Opposite and hom weak ω-categories

arXiv:2402.01611

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CT2024

Outline

- An novel definition of ω-categories and their computads
  - Joint with Dean, Finster, Reutter, Vicary
  - arXiv:2208.08719
- A construction of opposite ω-categories
- A lift of the suspension and hom adjunction
  - Alternative approach to Cottrell and Fujii 2022

What are weak ω-categories?

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- Globular sets are presheaves on the category of globes

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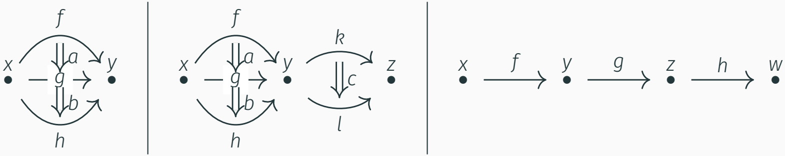
→ globular pasting diagrams

Globular Pasting diagrams

Globular pasting diagrams are the arities of the operations of ω-categories.



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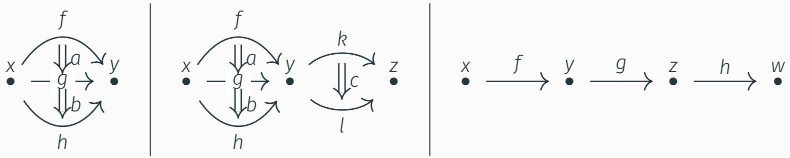
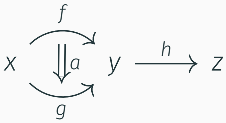
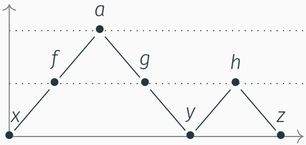


$x<f<a<g<y<h<z$



(a) Globular cardinals

(0,1,2,1,0,1,0)



(b) Zigzag sequences

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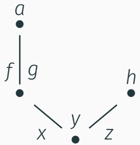

(c) Rooted, planar trees

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Contractible globular operads
(Batanin 1998; Leinster 2000)

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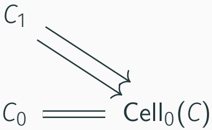

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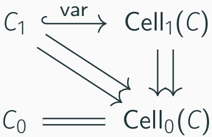

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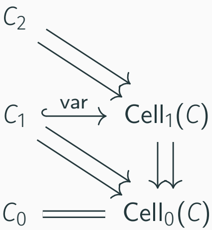

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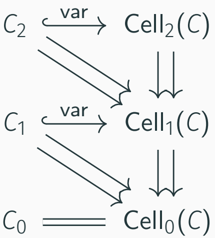

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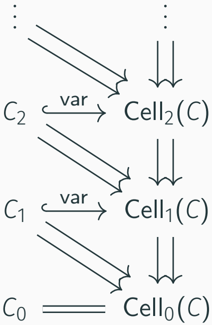

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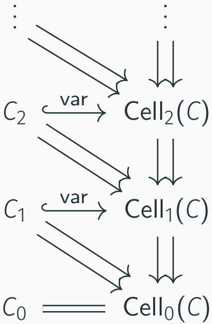

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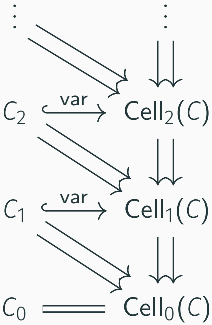

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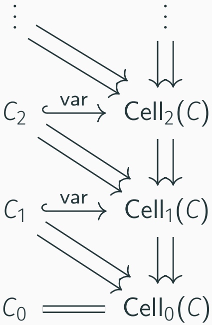

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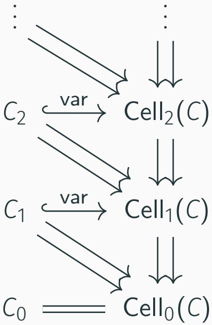

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An ω-category $\mathbb{X}$ is a globular set X equipped with operations

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⚠ Computads embed fully faithfully into ω-categories, allowing us to identify a computad with the ω-category it generates.

Opposites

Opposite globular sets

Recall that a globular set is a diagram of the form

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Given any subset $w \subseteq \mathbb{N}_{>0}$ of dimensions, we can define an involutive endofunctor

$$\text{op} \colon \text{gSet} \to \text{gSet}$$

by swapping the source and target maps of the dimensions in w.

Lifting op

Our goal is to lift the opposites functor to ω-categories.

$$\begin{array}{ccc} \omega\,\mathsf{Cat} & \overset{\mathsf{op}}{\dashrightarrow} & \omega\,\mathsf{Cat} \\ \downarrow & & \downarrow \\ \mathsf{gSet} & \xrightarrow{\mathsf{op}} & \mathsf{gSet} \end{array}$$

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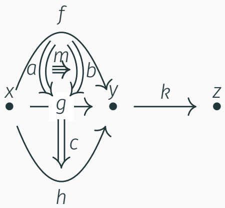
By the mate correspondence for the adjunction $\mathsf{op} \dashv \mathsf{op}$,

$$\iff \qquad \left\{ \begin{array}{c} \mathsf{op}' : \mathsf{op}(\mathsf{Cell}\,\mathsf{Free}) \Rightarrow (\mathsf{Cell}\,\mathsf{Free})\,\mathsf{op} \\ \text{compatible with unit and multiplication} \end{array} \right\}$$

Opposite pasting diagrams

Key observation: Globular pasting diagrams are closed under op.



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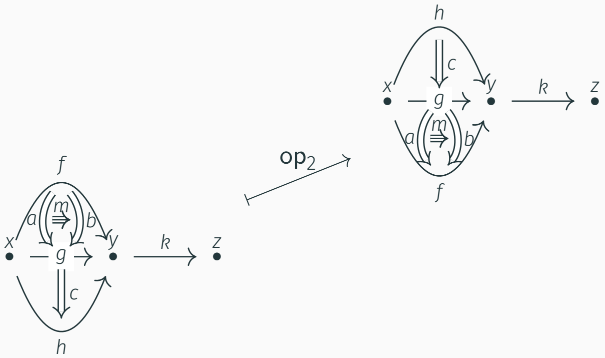

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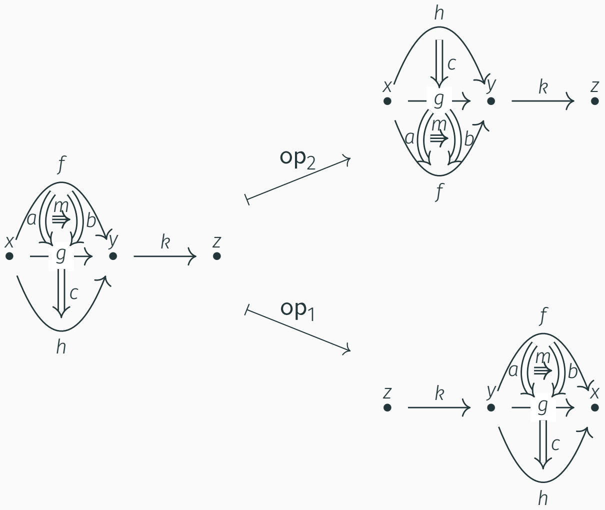

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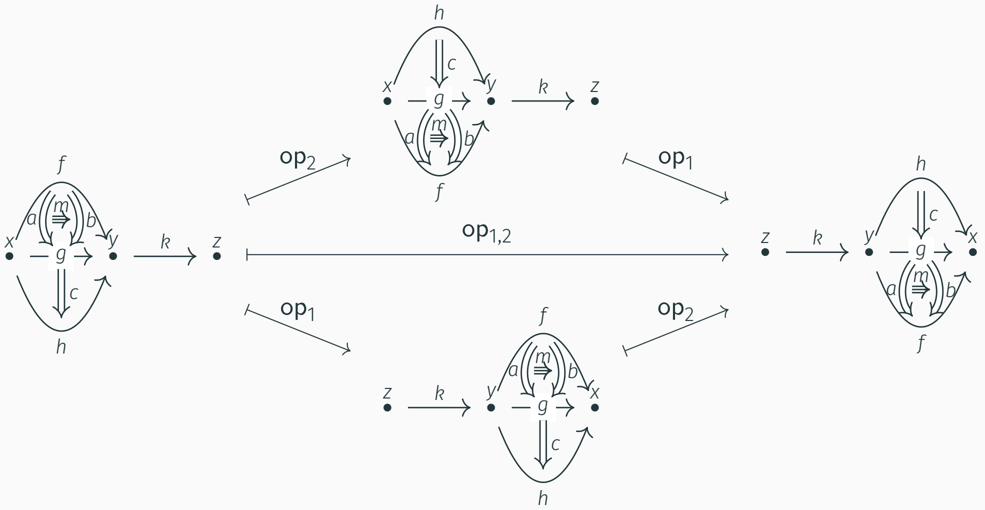

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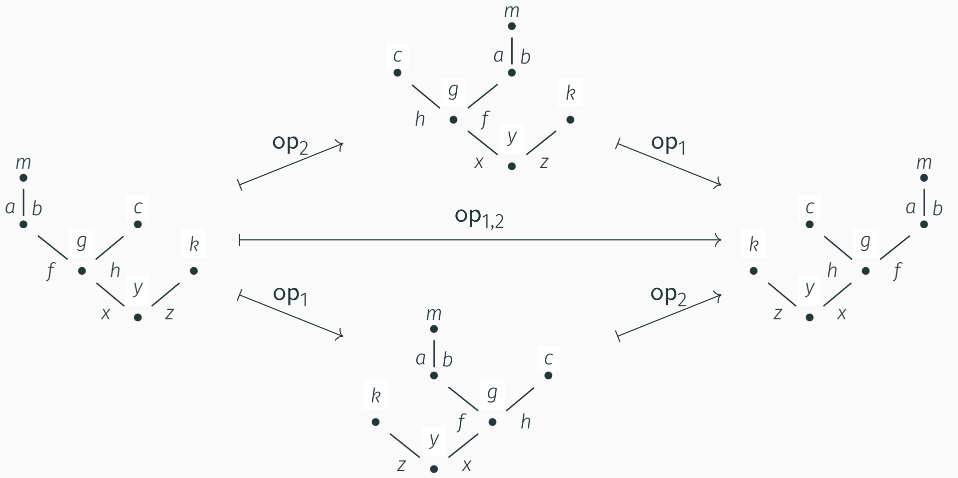

The construction of opposites

$$
\begin{array}{ccccc}
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{\scriptstyle \mathsf{op}}\uparrow & & {\scriptstyle \mathsf{op}}\uparrow & \nwarrow{\scriptstyle \mathsf{op}'} & \uparrow{\scriptstyle \mathsf{op}} \\
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\end{array}
$$

op and op′ are defined together by mutual structural induction

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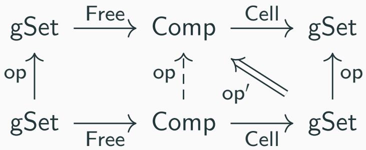

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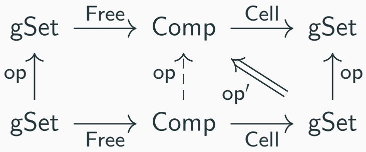

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- The natural transformation op′ is defined recursively by

$$\mathrm{coh}(\Gamma, u \to v, \sigma)^{\mathrm{op}} = \begin{cases} \mathrm{coh}(\Gamma^{\mathrm{op}}, u^{\mathrm{op}} \to v^{\mathrm{op}}, \sigma^{\mathrm{op}}) \\ \mathrm{coh}(\Gamma^{\mathrm{op}}, v^{\mathrm{op}} \to u^{\mathrm{op}}, \sigma^{\mathrm{op}}) \end{cases}$$

Suspension and hom

Suspension and hom globular sets

Definition
Globular sets are coinductively defined as graphs enriched in globular sets.

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The hom functor has a left adjoint, the suspension.

Lifting the adjunction

Our goal is to lift the opposites functor to ω-categories.

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Once we lift the right adjoint, we get a lift of the left adjoint as well (commuting with the free functors) by the adjoint lifting theorem (Johnstone 1975).

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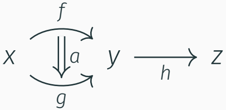

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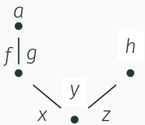

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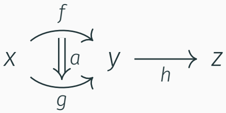

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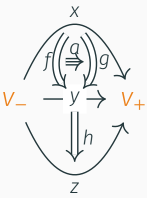
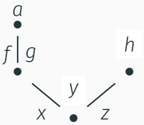
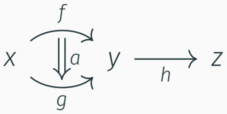




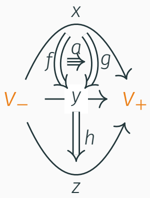
$\longmapsto$

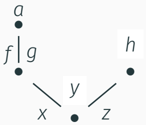
Suspending pasting diagrams

Key observation: Pasting diagrams are closed under the suspension functor

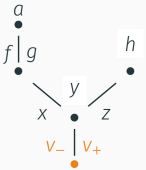


$\mapsto$

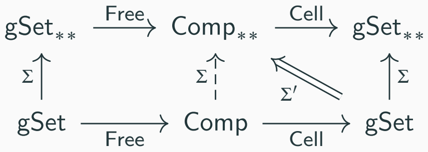




$\mapsto$

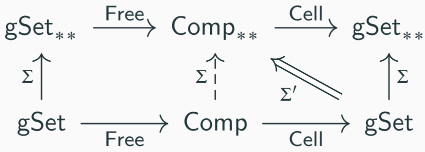


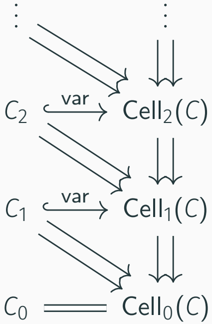
Suspending computads



Σ and Σ' are defined together by mutual structural induction

Suspending computads





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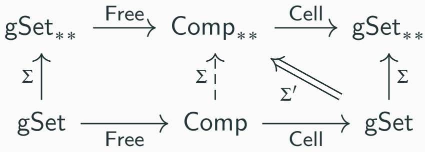
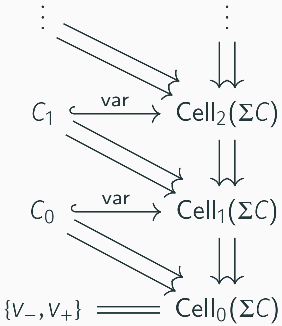
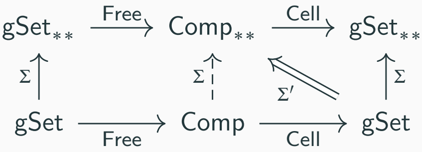


Σ and Σ′ are defined together by mutual structural induction

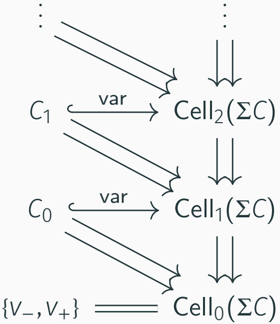


Source/Target is given by composition with Σ′

Suspending computads



Σ and Σ' are defined together by mutual structural induction



Source/Target is given by composition with Σ'

$$\Sigma'(\mathsf{coh}(\Gamma, u \to v, \sigma)) = \mathsf{coh}(\Sigma\Gamma, \Sigma' u \to \Sigma' v, \Sigma\sigma)$$

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model structure (co)limits

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